Hysteresis of the Hydraulic Jump Controlled by Sill in a Rectangular Channel

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ABSTRACT

A theoretical approach is proposed to compute the hysteresis of a controlled hydraulic jump by a broad-crested sill in a rectangular channel, when the inflow is generated by a gate and a standard spillway with an abrupt slope. The attention is focused on the calculation of both the maximum and minimum inflow Froude numbers compatible with the formation of the hydraulic jump in the energy dissipator stilling basin. When the inflow is generated by a gate, the theoretical development shows that the inflow Froude number is linked to the relative sill height by a cubic equation which can be solved using trigonometric functions. When the inflow is generated by a standard spillway with an abrupt slope, the inflow Froude number and relative sill are well-defined by an involutive function.

1. INTRODUCTION

Hydraulic jump can be controlled by a broad-crested sill[3,4,5,8] in order to ensure development of the jump by raising the upstream depth to control its position when changing the flow parameters and, therefore, contribute to a more compact stilling basin[3,10].

Previous studies of controlled hydraulic jumps have concerned rectangular channels[10,9,7], probably because of their simplicity and ease of implementation. It was intended primarily to define the functional relationship linking the various parameters influencing the phenomenon. The sill is considered to be broad when the surface profile is almost parallel to the rooftop as long as the sill length, L_0, is sufficient. The notion of a broad sill is also related to the characteristics of the overflow. According to the classification of Rao & Muralidhar[14] the sill is considered broad when $0.1 \leq (h - s)/L_0 < 0.35$, where h is the downstream depth and s is the sill height.

During various experimental tests it was found that hydraulic jump quickly disappears from the stilling basin when the inflow Froude number exceeds a limit value for a given sill height. It is governed either by the theory of the moving jump[12], or by the catastrophe theory[6]. However, this latter study showed the existence of a hysteresis range. The hydraulic jump reappears for an inflow Froude number much lower than that for which it existed. This is the hysteresis of the hydraulic jump which has never been theoretically or

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experimentally studied to date. Although discovered several years ago[1,2,13], the hysteresis of the controlled hydraulic jump has remained at the observation stage, especially controlled hydraulic jump by a broad-crested sill.

The main aim of this paper is to establish theoretical relationships which provide computation of the maximum and minimum inflow Froude numbers compatible with the formation of the hydraulic jump on the stilling basin using known hydraulic equations. The maximum inflow Froude number corresponds to the limit of the disappearance of the hydraulic jump, while the minimum inflow Froude number refers to the reappearance of the hydraulic jump on the stilling basin. Two configurations of the inflow are considered: the first corresponds to the inflow generated by a vertical sluice gate, while in the second the inflow is generated by an overflow dam.

2. CONFIGURATION WITH A SLUICE GATE

2.1 RELATIVE SILL HEIGHT

Figure 1 shows a hydraulic jump controlled by a broad-crested sill downstream of a sluice gate. The discharge is Q, upstream and downstream depths are h_1 and h_2 respectively, H_1 and H_2 are the energy head at the upstream and downstream sections of the jump, V_2 is the average downstream velocity, g is the acceleration due to gravity, ΔH is the energy head loss, and s is the height of the sill. The flow regime over the sill is critical, with a depth h_c, and total head H_c, provided by the following well-known equation:

$$H_c = \frac{3}{2} h_c \tag{1}$$

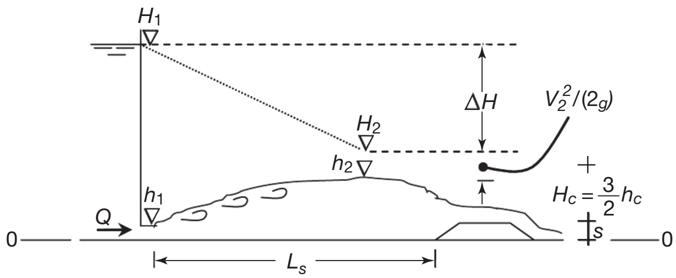

Figure 1. Hydraulic jump controlled by a broad-crested sill downstream of a sluice gate

The energy heads H_1 and H_2 are respectively given by the following relations:

$$H_1 = h_1 + \frac{q^2}{2gh_1^2} \tag{2}$$

$$H_2 = h_2 + \frac{q^2}{2gh_2^2} \tag{3}$$

where q is the discharge per unit width of the channel, which is given by:

$$q = \sqrt{gh_c^3} \tag{4}$$

Combining Equations 2, 3 and 4 leads to:

$$H_1^* = h_1^* + \frac{1}{2h_1^{*2}} \tag{5}$$

$$H_2^* = h_2^* + \frac{1}{2h_2^{*2}} \tag{6}$$

where $H_1^* = H_1/h_c$, $H_2^* = H_2/h_c$, $h_1^* = h_1/h_c$ and $h_2^* = h_2/h_c$.

Due to the supercritical flow in the upstream section of the hydraulic jump (Figure 1) the relative height, h_1^*, varies in the range $0 < h_1^* < 1$, while $h_2^* > 1$ in the downstream section of the hydraulic jump.

The relative head loss, $\Delta H^* = \Delta H/h_c$, can be written as:

$$\Delta H^* = H_1^*/H_2^* \tag{7}$$

By geometrical consideration, Figure 1 allows us to write:

$$\Delta H = H_1 - \left(s + \frac{3}{2} h_c\right) \tag{8}$$

which can also be rewritten as:

$$\Delta H^* = H_1^* - \left(\frac{s}{h_c} + \frac{3}{2}\right) \tag{9}$$

Introducing the relative sill height, $S = s/h_1$, into Equation 9 leads to:

$$\Delta H^* = H_1^* - \left(h_1^* S + \frac{3}{2}\right) \tag{10}$$

From Equation 10, the relative sill height is then:

$$S = \frac{H_1^* - \Delta H^* - 3/2}{h_1^*} \tag{11}$$

Taking into account Equation 7, Equation 11 is reduced to:

$$S = \frac{H_2^* - 3/2}{h_1^*} \tag{12}$$

Combining Equations 6 and 12 leads to:

$$S = \frac{h_2^*}{h_1^*} + \frac{1}{2h_2^{*2}h_1^*} - \frac{3}{2h_1^*} \tag{13}$$

It can be noted that $h_2^*/h_1^* = h_2/h_1$ corresponds to the sequent depth ratio of the hydraulic jump. According to Achour[1], h_2^* is related to h_1^* as:

$$h_2^* = \sqrt{(h_1^*/2)^2 + 2/h_1^*} - h_1^*/2 \tag{14}$$

For all practical purposes we can state that the relative height, h_1^*, varies in the range $0.2 \leq h_1^* \leq 0.4$[1], corresponding to an inflow Froude number varying in the wide range $4 \leq F_1 \leq 11$.

Although Equation 14 is explicit, it is possible to re-write it more simply in the following form, established in the practical range of h_1^*:

$$h_2^* = \sqrt{2/h_1^*} - h_1^*/2 \tag{15}$$

Maximum deviation between Equations 14 and 15 is about 0.4%. Introducing Equation 15 into Equation 13, one may write:

$$\left[S + \frac{1}{2} - 2\left(2\sqrt{2} - h_1^{*3/2}\right)^{-2}\right]h_1^{*3/2} + \frac{3}{2}h_1^{*1/2} - \sqrt{2} = 0 \tag{16}$$

Equation 16 shows that the relative sill height, S, depends solely on the relative inflow depth, h_1^*.

2.2 MAXIMAL INFLOW FROUDE NUMBER

In the configuration shown in Figure 1, hydraulic jump is in its extreme position corresponding to a maximum of both sill height $s = s_M$, and discharge $Q = Q_M$. If discharge (Q) exceeds maximum discharge (Q_M) hydraulic jump will move downstream, and a supercritical flow will be installed along the length of the stilling basin. For the same value of the relative sill height (S), experience shows that the reappearance of hydraulic jump only happens for a much lower value of the discharge. This phenomenon is known as hysteresis of the hydraulic jump, which many experiments have highlighted in the past.

In practice, the main problem encountered is the determination of relative inflow depth corresponding to both the extreme position of the hydraulic jump and maximum discharge (Q_M). This amounts to estimating the maximum inflow Froude number compatible with the presence of the hydraulic jump, as shown in Figure 1. This problem can be solved using Equation 16, provided the relative sill height is given.

Let us assume:

$$\zeta = 2\left(2\sqrt{2} - h_1^{*3/2}\right)^{-2} \tag{17}$$

In the wide practical range $0.2 \le h_1^* \le 0.4$, one may observe that $h_1^{*3/2} \le 2\sqrt{2}$ and, thus, $\zeta \approx 2\left(2\sqrt{2}\right)^{-2}$. Inserting this result into Equation 16 leads to:

$$\left(S + \frac{1}{4}\right)h_1^{*3/2} + \frac{3}{2}h_1^{*1/2} - \sqrt{2} = 0 \tag{18}$$

For a given value of relative sill height (S), deviation between Equations 16 and 18 depends on the relative height value (h_1^*). In the practical range $0.2 \le h_1^* \le 0.4$ the maximum error involved in h_1^* is about 1%, as can be seen in Figure 2.

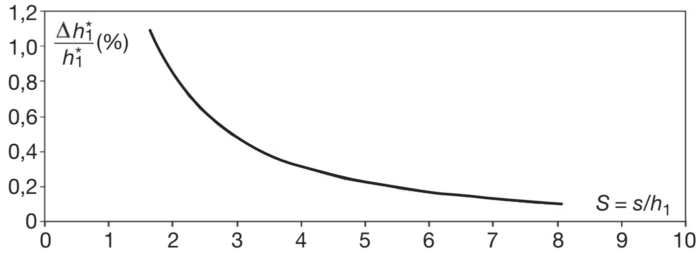

Figure 2. Deviation between Equations 16 and 18

Let us assume $X = h_1^{*1/2}$. Equation 18 can then be rewritten as:

$$\left(S + \frac{1}{4}\right)X^3 + \frac{3}{2}X - \sqrt{2} = 0 \tag{19}$$

or:

$$X^3 + \frac{3}{2\left(S + \frac{1}{4}\right)} X - \frac{\sqrt{2}}{\left(S + \frac{1}{4}\right)} = 0 \tag{20}$$

Equation 18 is therefore transformed into a reduced equation of the third degree. The discriminant is:

$$D = \left(\frac{t}{2}\right)^2 + \left(\frac{r}{3}\right)^3 \tag{21}$$

where $t = \frac{\sqrt{2}}{(S + 1/4)}$ and $r = \frac{3}{2\,(S + 1/4)}$. Since $S \geq 0$, the discriminant (D) is then strictly positive and the real root of Equation 20 can be written as:

$$X = 2\sqrt{\frac{r}{3}}\, sh(\beta/3) \tag{22}$$

or:

$$X = \sqrt{\frac{2}{S + 1/4}}\, sh(\beta/3) \tag{23}$$

The angle β is:

$$\beta = \mathrm{arg}sh\left(2\sqrt{S + 1/4}\right) \tag{24}$$

Taking into account the change of variables $X = h_1{}^{*1/2}$, Equation 23 is reduced to:

$$h_1^* = \frac{2sh^2(\beta/3)}{S + 1/4} \tag{25}$$

On the other hand, the maximum inflow Froude number is given by the following equation:

$$F_{1M} = \sqrt{\frac{q^2}{gh_1^3}} \tag{26}$$

Combining Equations 4 and 26, one may write:

$$F_{1M} = \sqrt{\frac{h_c^2}{h_1^3}} = h_1^{*-3/2} \tag{27}$$

Introducing Equation 25 into Equation 27 leads to:

$$F_{1M} = \frac{\sqrt{2}(S + 1/4)^{3/2}}{4sh^3(\beta/3)} \tag{28}$$

Bearing in mind that angle β is given by Equation 24, Equation 28 shows that the maximum inflow Froude number depends solely on the relative sill height (S).

2.3 EXAMPLE 1

For relative sill height $S = 5.55$, compute the maximum inflow Froude number (F_{1M}) compatible with the presence of controlled hydraulic jump in its extreme position, as shown in Figure 1.

Applying Equation 24, angle β is:

$$\beta = \text{arg}sh\left(2\sqrt{S + 1/4}\right) = \text{arg}sh\left(2 \text{ x } \sqrt{5.55 + 1/4}\right) = 2.27582906 \text{ radians}$$

According to Equation 28 the required value of the maximum inflow Froude number is then:

$$F_{1M} = \frac{\sqrt{2}(S + 1/4)^{3/2}}{4sh^3(\beta/3)} = \frac{\sqrt{2} \text{ x } (5.55 + 0.25)^{3/2}}{4 \text{ x } sh^3\ (2.27582906/3)} \approx 8.529$$

If the Froude number exceeds the computed value of F_{1M} the hydraulic jump will move downstream, and supercritical flow will take place along the length (L_s) of the stilling basin (Figure 1).

2.4 MINIMUM INFLOW FROUDE NUMBER OF THE REAPPEARANCE OF THE JUMP

Once hydraulic jump has disappeared for an inflow Froude number F_1 greater than F_{1M}, it reappears when reducing the discharge (Q). Figure 3 shows the state of the hydraulic jump reappearing for a minimal discharge (Q_m).

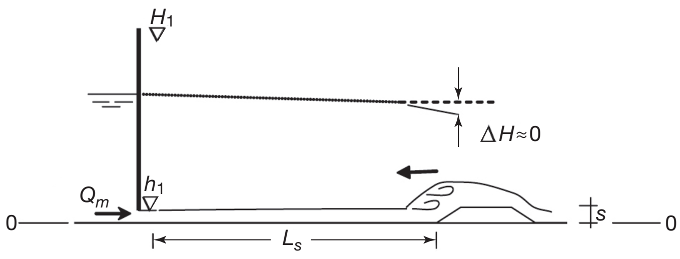

Figure 3. Definition sketch of the controlled hydraulic jump reappearing for a minimal discharge

At the limit of the reappearance of the hydraulic jump head loss is so small that one can write, from a theoretical point of view, that $\Delta H \approx 0$, implying $\Delta H^* = 0$.

Thus, Equation 11 is reduced to:

$$S = \frac{H_1^* - 3/2}{h_1^*} \tag{29}$$

The relative inflow depth (h_1*) of Equation 29 is related to a minimum inflow Froude number (F_{1m}) governing the reappearance state of the hydraulic jump.

Introducing Equation 5 into Equation 29 leads to:

$$S = 1 + \frac{1}{2h_1^{*3}} - \frac{3}{2h_1^*} \tag{30}$$

which can be rewritten as:

$$h_1^{*3} = \frac{3h_1^{*2}}{2(S-1)} - \frac{1}{2(S-1)} = 0 \tag{31}$$

This is a third degree equation which can be reduced without a second degree equation. By assuming the following change of variables – $h_1^* = x - a/3$ – one may obtain:

$$x^3 - rx + t = 0 \tag{32}$$

where $r = \frac{3}{4(S-1)^2}$, $t = \frac{1 - 2(S-1)^2}{4(S-1)^3}$.

The discriminant of Equation 32 is:

$$D = \left(\frac{t}{2}\right)^2 + \left(\frac{r}{3}\right)^3 = \frac{S(S-2)}{16(S-1)^4} \tag{33}$$

Thus, according to the range values of the relative sill height (S), the discriminant (D) can be negative, zero or positive. A particular study of Equation 33 and, hence, Equations 31 and 32 has shown that for:

i. $0 < S < 1$, the discriminant is negative, and the real root of Equation 31 is:

$$h_1^* = \frac{\cos(\beta + 60°) - 1/2}{S-1} \tag{34}$$

ii. $1 < S \leq 2$, the discriminant is negative or equal to zero, and the real root of Equation 31 is then:

$$h_1^* = \frac{\cos(\beta - 60°) - 1/2}{S-1} \tag{35}$$

Note that angle β in Equations 34 and 35 is given by the following equation:

$$\beta = \frac{1}{3}\cos^{-1}\left[1 - 2(S-1)^2\right] \tag{36}$$

iii. $S > 1$, the discriminant is positive, and the real root of Equation 31 is:

$$h_1^* = \frac{ch(\alpha) - 1/2}{S-1} \tag{37}$$

where angle α is governed by:

$$\alpha = \frac{1}{3}\operatorname{arg}ch\left[2(S-1)^2 - 1\right] \tag{38}$$

By analogy to Equation 27 the minimum inflow Froude number (F_{1m}) related to the relative height (h_1^*) of Equations 34, 35 and 37 is then:

$$F_{1m} = h_1^{*-3/2} \tag{39}$$

whence:

$$F_{1m} = \left[\frac{\cos(\beta + 60°) - 1/2}{S-1}\right]^{-3/2}, \ 0 < S < 1 \tag{40}$$

$$F_{1m} = \left[\frac{\cos(\beta - 60°) - 1/2}{S-1}\right]^{-3/2}, \ 1 < S \leq 2 \tag{41}$$

$$F_{1m} = \left[\frac{ch(\alpha) - 1/2}{S - 1}\right]^{-3/2}, S > 2 \tag{42}$$

As can be seen, F_{1m} depends solely on the relative sill height (S). Note that in the practical range $0.2 < h_1^* < 0.4$, the relative sill height is strictly greater than unity. Thus, the phenomenon is rarely governed by Equation 40.

2.5 EXAMPLE 2

Take the data of Example 1 and determine for the same value of the relative sill height S = 5.55, the value of the minimum inflow Froude number corresponding to the beginning of the reappearance of the controlled hydraulic jump.

The imposed relative sill height is greater than 2, thus the phenomenon is governed by both Equations 38 and 42. Angle α is then:

$$\alpha = \frac{1}{3}\arg ch\left[2(S - 1)^2 - 1\right] = \frac{1}{3}\arg ch\left[2 \times (5.55 - 1)^2 - 1\right] = 1.46398252 \text{ radians}$$

According to Equation 42, the required minimum inflow Froude number (F_{1m}) is:

$$F_{1m} = \left[\frac{ch(\alpha) - 1/2}{S - 1}\right]^{-3/2} = \left[\frac{ch(1.46398252) - 1/2}{5.55 - 1}\right]^{-3/2} = 4.096$$

3. CONFIGURATION WITH AN OVERFLOW DAM

3.1 MAXIMUM INFLOW FROUDE NUMBER

Figure 4 is a schematic representation of a controlled hydraulic jump by a broad-crested sill, generated by a supercritical flow on the downstream face of an overspill dam. The hydraulic jump is in its extreme position, corresponding to a maximum inflow Froude number F_{1M}.

The flow is characterized by the critical head (H_c) on the sills, and the inflow depth (h_1) at the toe of the overspill dam, which corresponds also to the inflow depth of the hydraulic jump on the stilling basin of length L_s, and downstream depth h_2. The height of the overspill dam and the broad-crested sill are represented by s_1 and s_2, respectively.

Total head upstream of the hydraulic jump is H_1, whereas H_2 is the total head at the downstream section. The difference between H_1 and H_2 corresponds to the head loss (ΔH) caused by the hydraulic jump. The head loss due to friction on the downstream face of the overspill dam is represented by h_f, which cannot currently be estimated. However, in all cases the friction loss (h_f) is relatively small when compared to the total head (H_1).

On the other hand computation of the inflow depth (h_1), taking into account head loss (h_f) in a steeply-sloping case, has been the subject of a particular study by Houichi & Achour[11].

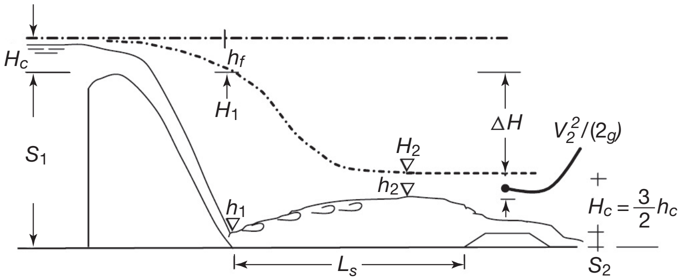

Figure 4. Hydraulic jump controlled by a broad-crested sill generated by an overflow dam

Neglecting the head loss (h_f), Figure 4 allows one to write:

$$H_1 = s_1 + \frac{3}{2}h_c \tag{43}$$

$$H_2 = s_2 + \frac{3}{2}h_c \tag{44}$$

Let us assume $\Delta s = s_1 - s_2$. The head loss of the hydraulic jump $\Delta H = H_1 - H_2$ is then $\Delta H = \Delta s$, which can be rewritten as:

$$\frac{\Delta s}{s_1} = \frac{\Delta H}{s_1} \tag{45}$$

Combining Equations 43 and 45 leads to:

$$\frac{\Delta s}{s_1} = \frac{\Delta H}{H_1 - 3h_c/2} \tag{46}$$

Equation 46 then becomes:

$$\frac{\Delta s}{s_1} = \frac{\Delta H^*}{H_1^* - 3/2} \tag{47}$$

Referring to Equations 5, 6, 7, 14 and 47 one may write that the relative inflow depth, h_1^*, depends solely on the relative inflow depth, $\Delta s/s_1$. Figure 5 shows the variation of $\Delta s/s_1$

with respect to $h_1{}^*$ corresponding to the solid line. In the same figure the second bisecting line, which is governed by $\Delta s/s_1 + h_1{}^* = 1$, corresponds to the dashed line.

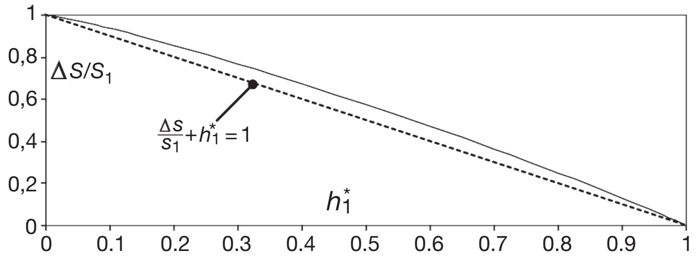

Figure 5. Variation of $\Delta s/s_1$ *vs* $h_1{}^*$.
(-) Equation 47; (- - -) second bisecting line

A particular study of both Figure 5 and Equation 47 reveals the following results:

i. $h_1{}^* \to 1$ when $\Delta s/s_1 \to 0$;
ii. $h_1{}^* \to 0$ when $\Delta s/s_1 \to 1$;
iii. The curve $\Delta s/s_1 = f(h_1{}^*)$ is above the second bisecting line;
iv. The curve $\Delta s/s_1 = f(h_1{}^*)$ is governed by $(\Delta s/s_1)^m + h_1{}^{*n} = 1$, where $m > 1$ and $n > 1$;
v. It is observed that $(\Delta s/s_1)_i = f[(h_1{}^*)_j]$ and $(h_1{}^*)_j = \mathrm{f}\,[(\Delta s/s_1)_i]$. This important result allows us to conclude that function f is involutive, implying $m = n$;
vi. The exponents m and n are as $m = n \approx 9/8$.

Thus, $\Delta s/s_1 = f(h_1{}^*)$ is an involutive function described by the following relationship:

$$(\Delta s/s_1)^{9/8} + h_1^{*9/8} = 1 \tag{48}$$

Introducing Equation 27 into Equation 48, and rearranging, leads to:

$$F_{1M} = \left[1 - \left(\frac{\Delta s}{s_1}\right)^{9/8}\right]^{-4/3} \tag{49}$$

In practice, the known parameters are the sill heights s_1 and s_2. Thus, Equation 49 allows direct determination of the maximum inflow Froude number compatible with the complete formation of the controlled hydraulic jump, as presented in Figure 4. However, for accuracy reasons, and in order to take into account the effect of friction loss on the downstream face of the overspill dam, we strongly recommend the use of Equation 49 for $\Delta s/s_1 \geq 0.6$, which encompasses a wide range of practical cases.

3.2 EXAMPLE 3

The energy dissipation structure presented in Figure 4 is characterized by s_1 = 14m and s_2 = 2.6m. Compute the maximum inflow Froude number compatible with the complete formation of the hydraulic jump on the stilling basin:

$$\frac{\Delta s}{s_1} = \frac{s_1 - s_2}{s_1} = \frac{14 - 2.6}{14} = 0.81428571$$

According to Equation 49, the required maximum inflow Froude number is:

$$F_{1M} = \left[1 - (\Delta s/s_1)^{9/8}\right]^{-4/3} = \left(1 - 0.81428571^{9/8}\right)^{-4/3} \simeq 8.2$$

If the inflow Froude number exceeds the computed value of F_{1M}, by increasing the discharge the hydraulic jump will move downstream, disappearing quickly from the stilling basin. A supercritical flow will take place along the length of the stilling basin, and the hydraulic jump will reappear only for an inflow Froude number F_{1m}, which is much lower than F_{1M}. Computation of F_{1m} is carried out in the same way as described in Section 2.4 for the sluice gate configuration.

4. CONCLUSIONS

The study contained within this paper has been devoted to the hysteresis of a controlled hydraulic jump by a broad-crested sill considering two flow configurations. The first corresponds to an inflow generated by a sluice gate, whereas in the second the inflow is generated by an overflow dam.

The main objective was to establish a theoretical approach defining the functional relationship linking the various parameters influencing the phenomenon. In the sluice gate configuration the study was able to establish non-dimensional relationships for the computation of the relative sill height, and maximum and minimum inflow Froude numbers compatible with the formation of the hydraulic jump on the stilling basin. The relations obtained were of the third degree, whose resolution was possible with the aid of trigonometric functions.

In the configuration with an overflow dam, it was clearly established that the relative inflow depth is related to the relative sill height of the overspill dam by an involutive function.

Several numerical illustrations have also been presented in order to explain the procedure of calculation.

REFERENCES

[1] Abecasis, F & Quintela, A, 'Problems of Hydraulic Hysteresis on Steady Free Surface Flow', *Proceedings*, 9th General Meeting, International Association for Hydraulic Research, Dubrovnik, Croatia (1961).

[2] Abecasis, F & Quintela, A, 'Hysteresis in Steady Free-surface Flow', *International Water Power & Dam Construction*, April, pp147-157 (1964).

[3] Achour, B, 'Energy Dissipators by Hydraulic Jump', *PhD thesis*, University of Tizi-Ouzou, Algeria (1997).

[4] Achour, B & Debabèche, M, 'Hydraulic Jump Controlled by Sill in a U Shaped Channel', *Journal of Hydraulic Research*, Vol 41, Issue 1, pp97-103 (2003a).

[5] Achour, B & Debabèche, M, 'Control of Hydraulic Jump by Sill in a Triangular Channel', *Journal of Hydraulic Research*, Vol 41, Issue 3, pp319-325 (2003b).

[6] Austria, P M, 'Catasptrophe Model for the Forced Hydraulic Jump', *Journal of Hydraulic Research*, Vol 25, Issue 3, pp269-280 (1987).

[7] Bradley, J N & Peterka, A J, 'The Hydraulic Design of Stilling Basins: Hydraulic Jumps on a Horizontal Apron', *Proceedings*, ASCE, Journal of the Hydraulic Division, Paper 1401, Vol 83, pp61-78 (1957).

[8] Debabèche, M & Achour, B, 'Effect of Sill in a Triangular Channel', *Journal of Hydraulic Research*, Vol 45, Issue 1, pp135-139 (2007).

[9] Forster, J W & Skrinde, R A, 'Control of the Hydraulic Jump by Sills', Trans., ASCE, 115, pp973-1022 (1950).

[10] Hager, W H & Li, D, 'Sill-controlled Energy Dissipater', *Journal of Hydraulic Research*, Vol 30, Issue 2, pp165-181 (1992).

[11] Houichi, L & Achour, B, 'Flow Depth Computation at the Toe of an Overflow Dam in Steeply-sloping Case', *Dam Engineering*, Vol XVII, Issue 4, pp245-256 (2007).

[12] Martin Vide, J P, Dolz, J & Del Estal, J, 'Kinematics of the Moving Jump', *Journal of Hydraulic Research*, Vol 31, Issue 2, pp171-186 (1993).

[13] Quintela, A & Abecasis, F, 'Hysteresis in the Transition from Supercritical to Subcritical Flow', Laboratório Nacional de Engenharia Civil, Memoria Nr. 523, Lisbon, Portugal (1979).

[14] Rao, N S G & Muralidhar, D, 'Discharge Characteristics of Weirs of Limit Crest Width', *La Houille Blanche*, Vol 18, pp537-545 (1963).

Notations

F_1 = Inflow Froude number (-)
F_{1m} = Minimum inflow Froude number (-)
F_{1M} = Maximum inflow Froude number (-)
g = Acceleration due to gravity (ms^{-2})
h_1 = Inflow depth (m)
h_2 = Downstream depth (m)
h_c^* = Critical depth (m)
h_1^* = Relative inflow depth (-)
h_2 = Relative downstream depth (-)
H_1 = Upstream total head (m)
H_2 = Downstream total head (m)
H_1^* = Relative upstream head (-)
H_2^* = Relative downstream head (-)
Q = Discharge (m^3s^{-1})
q = Discharge per unit width of rectangular channel (m^2s^{-1})
s = Sill height (m)
S = Relative sill height (-)
V_1 = Mean upstream velocity (ms^{-1})
V_2 = Mean downstream velocity (ms^{-1})