COMPUTATIONAL PERFORMANCE COMPARISON BETWEEN FVM AND FEM FOR 3D MAGNETOSTATIC PROBLEMS

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Abstract: The present paper deals with the comparison of the computational performance between the finite volume FVM and the finite element FEM methods. Under the same mesh size, a practical 3D magnetostatic problem proposed by IEEJ has been resolved. Detailed discretization and computational data are provided. Compared to FEM, finite volume method has the advantage to reach the solution with reduced number of iterations and CPU time.

Keywords: finite volume method, finite element method, 3D magnetostatic.

1. Introduction

Over the past decades, various computational techniques have been developed to solve electromagnetic problems. Generally, the finite element FEM is the most method which comes into view. In the fact, it has been demonstrated, over large number of papers, to be useful and powerful tool in magnetic field computation.

Several researches are interested to compare the FEM with other numerical methods. For example in [1] a detailed comparison, concerning the formulation and the way how the unknown is approached on the mesh grid between FEM and other numerical methods except FVM, is given.

Now widely used in various applications, the finite volume method FVM is one of the numerical methodologies applied in computational of fluid dynamic problems. It has also been efficiently applied for modeling electromagnetic problems. For example in [2], the method has been successfully introduced to solve a particular 2D magnetostatic problem.

Few critical reviews exist which compare FVM with other numerical methods. The task of citing these papers is an easy one such as [3] and [4]. In [4] the difference between the FVM method and the FEM method on elements constructing is presented, it is shown that accurate estimation of the potential distribution can be obtained with an FVM solution.

In this paper, the FVM will be compared with the FEM for the solution of a practical 3D magnetostatic problem proposed by IEEJ. A detailed comparison, including the matrix sparsity, the precision of the results and computational performance, is carried out under the same mesh size. The result shows that the FVM is a rapid tool for the magnetic field calculation.

2. FVM and FEM systems of equations

The aim of this paper is to give an idea about the difference between FVM and FEM in terms of computational performance. Therefore, the attention is focused on the solution of the algebraic systems of equations resulting from the discretization. Theory and implementation are well exist in literature such as [5] for the finite volume method.

Let us consider the magnetostatic problem described in the next section (Fig.3). Making use of the magnetic vector potential $\boldsymbol{A}$ and the Coulomb gauge, the considered equation with Dirichlet conditions are:

$$\nabla \times (\nu.\nabla \times \boldsymbol{A}) - \nabla(\nu.\nabla.\boldsymbol{A}) = \boldsymbol{J}_s \quad in\ \Omega \tag{1}$$

$$\boldsymbol{A} = 0 \quad in\ \Gamma \tag{2}$$

where $\boldsymbol{J}_s$ and ν are respectively, the source current density and magnetic reluctivity. Under the same mesh size, the problem geometry Ω is divided into a large number of hexahedron and tetrahedron elements by means of FVM and FEM techniques, respectively (Fig.1).

(*a*)-*Tetrahedral element* (*FEM*) (*b*)-*Hexahedral element* (*FVM*)
Figure 1: FVM and FEM elements

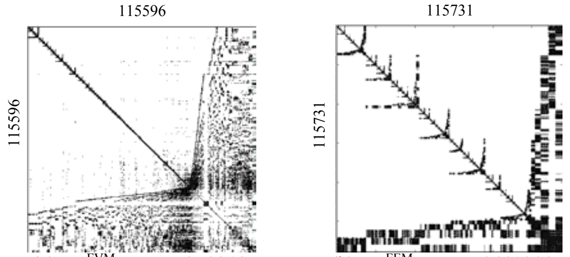
By process with the discretization steps of each method, equation (1) leads to two algebraic systems of equations:

$$\left[\mathrm{K}^{\mathrm{FVM}}\right]\cdot\left[\boldsymbol{A}\right] = \left[\mathrm{b}^{\mathrm{FVM}}\right] \tag{3}$$

$$\left[\mathrm{K}^{\mathrm{FEM}}\right]\cdot\left[\boldsymbol{A}\right] = \left[\mathrm{b}^{\mathrm{FEM}}\right] \tag{4}$$

where $\mathrm{K}^{\mathrm{FVM}}$ and $\mathrm{K}^{\mathrm{FEM}}$ are the global assembled matrices obtained with FVM and FEM, respectively. $\mathrm{b}^{\mathrm{FVM}}$ and $\mathrm{b}^{\mathrm{FEM}}$ are the source vectors. After Cuthill-McKee ordering, matrices sparsity structures are shown in Fig.2. It is clear that $\mathrm{K}^{\mathrm{FVM}}$ with 870212 nonzero entries (nnz) has the sparest structure versus $\mathrm{K}^{\mathrm{FEM}}$ which contain 4641209 nonzero entries.

Since (3) and (4) are large and spare, iterative methods are recommended. The Jacobian Conjugate Gradient JCG solver, which uses the diagonal of the global matrix as a preconditioner, was selected.



(a) -K ^{FVM} , *nnz* = 870212 *(b)*- K ^{FEM} , *nnz* = 4641209

Figure 2: Matrices sparsity structure

3. FVM and FEM results

The target domain of the chosen problem for the comparison is shown in Fig.3. It has a simple iron block and exciting coil. The relative permeability of the iron core is assumed 1000. The magnetomotive force is 3000 AT. Measured values of the magnetic flux density are given in [6].

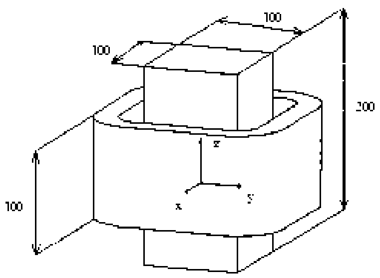
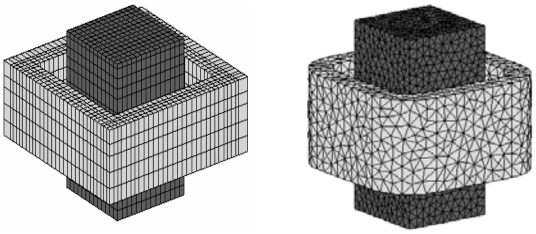


Figure 3: IEEJ 3D magnetostatic Problem

Under the same size, Fig.4.a and Fig.4.b shows the mesh of the whole domain constructed with FVM and FEM, respectively.



(a)-FVM mesh *(b)-FEM mesh*

Figure 4: Discretization of the computational domain

Measured and computed values of the *z* component of the magnetic flux density (***Bz***) are compared in Fig.5. Discretization and computational data are given in Table 1. The calculations have been carried out on PC with 1 GB RAM and 3 GHz processor.

From the results, FVM required:

- 2.5 times less iterations than FEM
- 7.15 times less CPU time than FEM

Furthermore, concerning the nnz entries, the FVM required less memory capacity than FEM. Accordingly; we can confirm that the finite volume method is a useful tool for large 3D problems.

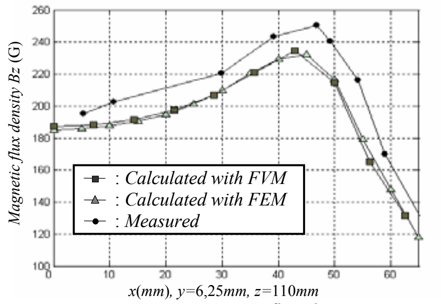


Figure 5: Magnetic flux density

Comparison points		FEM	FVM
Error (%)	Maximum	9.02	9.78
	Average	6.60	6.57
CPU time (sec)	System construction	63	6
	Solution	273	41
	Total	336	47
System size	No. Nodes	38577	38532
	No. Equations	115731	115596
	Nnz(K)	4641209	870212
Number of Iterations		435	174

Table 1: Discretization and computational data

4. Conclusion

Under the same mesh size, FVM and FEM have been applied for the solution of a 3D magnetostatic problem. Almost, an identical accuracy was obtained. Compared to FEM, the FVM saves much computational time and makes the requirement of the computer memory capacity smaller.

Work context

The present work is carried out within an Algerian-French cooperation PAI-Tassili program.

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