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Fast Motion Estimation Algorithm Based on Complex Wavelet Transform

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Abstract In this paper, we introduce an algorithm for motion estimation. It combines complex wavelet decomposition and a fast motion estimation method based on affine model. The principle of wavelet transform is to decompose hierarchically the input image into a series of successively lower resolution reference images and detail images which contain the information needed to be reconstructed back to the next higher resolution level. The motion estimation determines the velocity field between two successive images. This phase can be extracted from this measure descriptive information of the sequence. Motion Estimation (ME) is an important part of any video compression system, since it can achieve significant compression by exploiting the temporal redundancy existing in a video sequence. This paper described a method from calculating the optical flow of an image sequence based on complex wavelet transform. It consists to project the optical flow vectors on a basis of complex-valued wavelets. Thus, we add an additional assumption on the shape of the velocity field that we want to find, which is the affinity of the optical flow. The two-dimensional affine motion model is used to formulate the optical flow problem by coarse resolution simultaneously coarse-and-fine, beside the traditional approach by coarse-to-fine, to avoid the error propagation during the decomposition of coarse level to fine level. This method opens the way for a quick and low-cost computing optical flow.

Keywords Motion estimation · Complex wavelet · Fast two frame algorithm · Coarse and fine model

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1 Introduction

Today, the use of Wavelet Transforms (WT) is becoming ubiquitous in signal processing owing to the power of the multi-resolution technique and the existence of fast algorithms. Research on wavelets mainly concerns real-valued wavelet bases and filter banks as evident by the excess of publications on the subject. However, complex wavelet bases and filter banks have been seldom discussed.

Both the Continuous Wavelet Transform (CWT) and Fourier Transform are defined for signals that are complex as well as real. The basis functions of the Fourier transform are the complex exponential functions, the Short Time Fourier transform (STFT) is equivalent to implementing a bank of band-pass complex-valued filters having equal band widths. The mother wavelets of CWT are often complex valued functions, such as the Morlet wavelet and the Gabor wavelet, etc. Present theory and techniques have already shown that these approaches are better able to handle complex signals. Therefore, adopting complex filter banks and associated wavelet bases for similar applications is a natural choice.

Optical Flow (OF) estimation is an essential problem in motion analysis of image sequences [1].

It provides information needed for video technology, such as object tracking, image segmentation, and motion compensation. A Great number of approaches for OF estimation have been proposed in the literature, including gradient-based, correlation-based, energy based, and phase based techniques [2–4]. Each approach has its own merits and detractions depend on the intended use and characteristics of imagery. An extensive review of the performance's optical flow methods can be found in [1, 5, 6].

In the recent years, the Discrete Wavelet Transform (DWT) has been used for motion estimation in a number of ways. In [7], they proposed a fast motion algorithm based on real wavelet transform, their approach requires only two frames. The use of real-valued coefficients of wavelet makes the transform shift variant [8–10] and does not enjoy the directionality of complex wavelet transform. In this paper, we perform their method by combining the complex wavelet transform and fast two frame motion estimation algorithms.

The rest of the paper is organized as follows. In Section 2, we introduce the principle and the description of the proposed algorithm. Section 3 shows the experimental performance of optical flow estimation. Finally, Section 4 concludes our contribution and merits of this work.

2 Optical Flow Estimation Algorithm Based on Complex Wavelet Transform

2.1 Fast Optical Flow Estimation

A basic model of motion assumes that the brightness signal translates with constant velocity and direction. In such case we can write:

$$I(x,t) = I(x-v,0) \tag{1}$$

Where *I(x,t)* the brightness intensity of the image x = (*x*, *y*) is position in the image and v = *(u,v)* is optical flow.

The differential methods use the following gradient constraint equation:

$$(I_x(\mathrm{x},t), I_y(\mathrm{x},t)).\mathrm{v} + I_t(\mathrm{x},t) = 0 \tag{2}$$

Where $I_{x[y][t]}$ (.) is the partial derivative of $I(x,t)$ with respect (x,y) or t..

To handle the aperture problem, we assume that the local optical flow v(u,v) satisfies the following affine model:

$$\begin{bmatrix} u \\ v \end{bmatrix} = \begin{bmatrix} p_1 & p_2 \\ p_4 & p_5 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} + \begin{bmatrix} p_3 \\ p_6 \end{bmatrix} \tag{3}$$

The major problem of the traditional coarse-to-fine method is the error propagation. To overcome this inconvenient, the optical flow is computed in a coarse-and-fine manner. The last one is estimates at all scale levels simultaneously. For a good review, see [7].

Mathematically, in the $m \times n$ neighborhood S of the pixel (x_i,y_i) in the original image $(i=1,....M,$ and $M = m \times n)$ Eq. (3) can be rewritten as

$$\mathrm{u} = f(X,Y)p \tag{4}$$

Where X and Y are x- and y-coordinate matrices and $p = (p_1,p_2,p_3,p_4,p_5,p_6)^T$ is the vector of affine model parameters. The function $f(,)$ is defined as follows:

$$f(X,Y) = \begin{bmatrix} & \cdots & & & \cdots & \\ x_i & y_i & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & x_i & y_i & 1 \\ & \cdots & & & \cdots & \end{bmatrix}^T \tag{5}$$

By combining Equations (2) and (4)

$$I_X.f(X,Y)p = -I_t \tag{6}$$

Where

$$I_X = \begin{bmatrix} I_{x_1} & I_{y_1} & 0 & 0 & \cdots & 0 & 0 \\ \vdots & \vdots & \vdots & \vdots & \ddots & 0 & 0 \\ 0 & 0 & 0 & 0 & \cdots & I_{x_M} & I_{y_M} \end{bmatrix} \tag{7}$$

$$I_t = (I_{t_1}, I_{t_2}, \ldots I_{t_M})^T \tag{8}$$

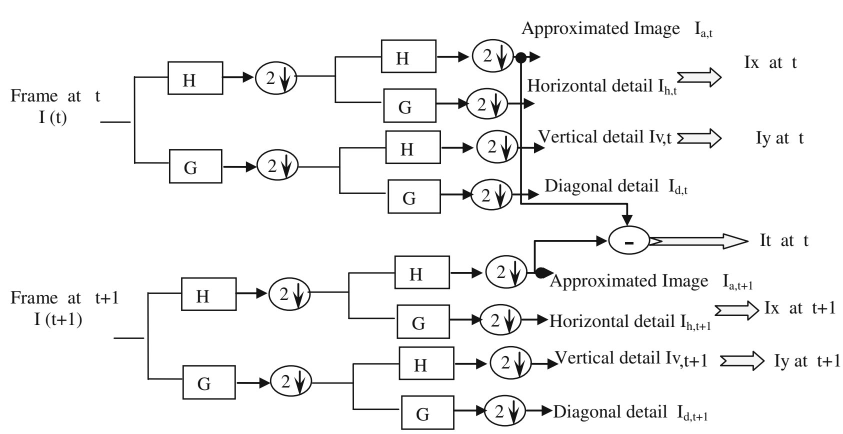


Figure 1 Computing the two spatial derivative Ix, Iy and the temporal derivative It in the wavelet basis, where H is the low-pass filter and G is the high-pass filter.

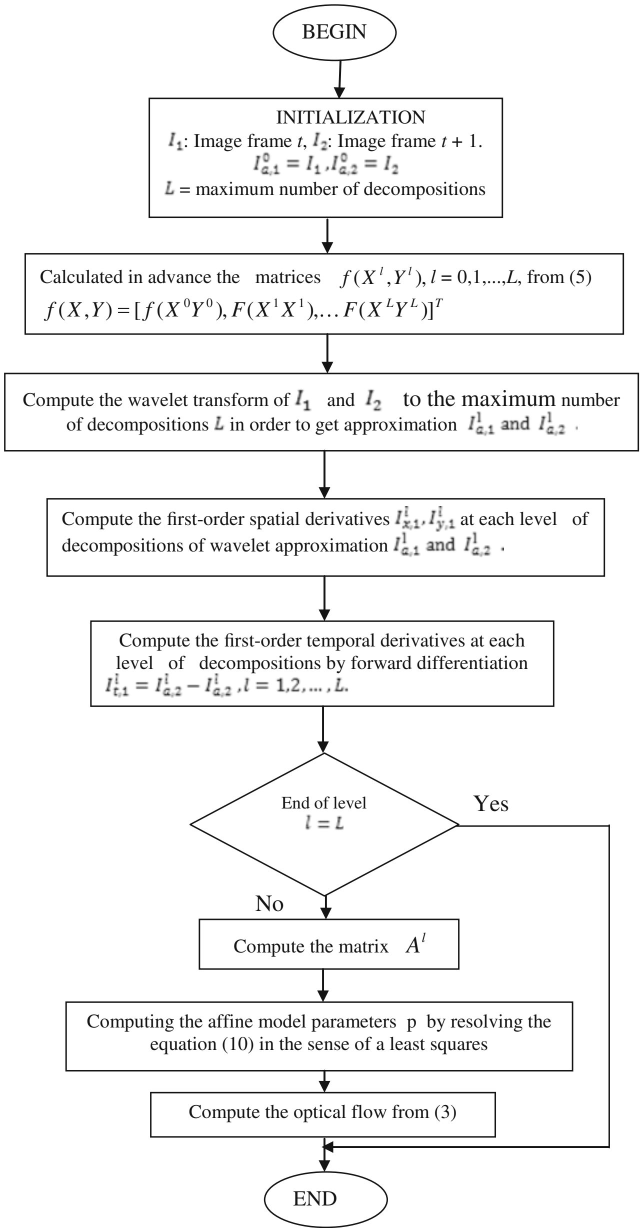


Figure 2 The proposed method of optical flow estimation.

By applying the wavelet transforms at the level l, we get the hierarchical gradient constraint functions (see Fig. 1).

$$A^l p = -I_t^l \tag{9}$$

The global equation of gradient constraint at all scale levels L is:

$$\begin{aligned} &Ap = b \\ &A = [A^0, A^1, \dots, A^L]\ b = -\left(I_t^0, I_t^1, \dots I_t^L\right)^T \end{aligned} \tag{10}$$

2.2 Complex Wavelet Transform

The suggested method is based on exploiting the complex wavelet transform in motion estimation. A real wavelet basis was used for fast two frame motion estimation algorithm in [7]. However, the transform used does not have the shift invariance and directional selectivity properties [8–10] of the Complex wavelet bases.

The directional propriety of complex wavelet offers a potential advantage in motion estimation, which makes the algorithm able to estimate, with accuracy, many kinds of movement (translations, rotation, zoom….).

The proposed method is based on complex daubechies wavelets obtained from a complex multiresolution analysis. The complex daubechies wavelets are similar to the DWT, but uses complex-valued filters (see Fig. 1). The mother wavelets of CWT are often complex-valued functions.

In addition to sharing the same properties as the real daubechies wavelets (i.e. compact support, orthogonality, and vanishing moments) are symmetric they have a better interpolation capability due to identical vanishing moments.

2.3 Algorithm

We implemented a coarse-and-fine approach based on complex wavelet transform to compute the velocity field. By applying the complex wavelet transforms at the level l, we project the hierarchical gradient constraint functions on a basis of complex- daubechies wavelets.

Table 1 Comparison between different methods of sequence « Translating Tree ».

Methods	Images	Average error	Deviation	Density (%)
Motion estimation using complex wavelet	2	1.20°	3.70°	100
Motion estimation using real wavelet [7]	2	3.67°	2.18°	100
Motion estimation using real wavelet(our simulation)	2	5.29°	3.98°	100
Horn & Schunk (original)	2	38.43 °	27.67°	100
Horn & Schunk (modified)	7–13	2.02°	2.27°	100
Anandan	2	1.25°	3.10°	100
Singh	2	1.25°	3.29°	100

Table 2 Comparison between different methods of sequence « Diverging Tree ».

Methods	Images	Average error	Deviation	Density (%)
Motion estimation using complex wavelet	2	0.79°	0.68°	100
Motion estimation using real wavelet [7]	2	1.67°	0.88°	100
Motion estimation using real wavelet(our simulation)	2	5.35°	4.56°	100
Horn& Schunk (original)	2	12.02°	11.72°	100
Horn et Schunk (modified)	7–13	2.55°	3.67°	100
Anandan	2	7.64°	4.96°	100
Singh	2	8.60°	4.78°	100

In order to compute the motion field, the global equation of gradient constraint at all scale levels L is resolved in order to find the parameters of the affine model.

The Fig. 2 illustrates the main stages of the optical flow estimation.

3 Experiments Results

In our experiments, we used three sets of results obtained using four different methods for comparison, and the proposed complex wavelet-based method and real wavelet based method.

For our simulation, the low and high pass filters of real wavelets are done by: h_{low}=[0.5, 0.5]; h_{high}=[0.5, −0.5], and the used complex daubechies wavelets and its filter bank with low and high pass filters are done by: $h_{low} = [1-j, 4-j, 4+j, 1+j]/10$; $h_{high} = [-1-2j, 5+2j, -5+2j, 1-2j]/14$.

We evaluated the optical flow by using the angular error measurement between the correct velocity (u_c,v_c) and the estimated velocity (u_e,v_e) with 100 % density [7].

$$\theta_{err} = arccos\left(\frac{u_e u_{c+} v_e v_c + 1}{\sqrt{u_e^2 + v_e^2 + 1}\sqrt{u_c^2 + v_c^2 + 1}}\right)$$

The average error and standard deviation of θ_{err} were calculated.

Three synthetic image sequences were used to test our algorithm quantitatively and compared with other optical flow techniques:

Translating Tree This sequence simulates translational camera motion along the X axis

Diverging Tree This sequence simulates a 3 synthetic camera moving toward a planar image of a tree. The speeds range from 0 in the middle (at the focus of expansion) to 1.4 pixels/frame on the left and 2.0 pixels/frame on the right.

Yosemite This is a more complex test sequence with a wide range of velocities, occluding edges, and severe aliasing in the lower portions of the images.

For the three sequences usually used, Yosemite Translating and Diverging Tree. The Tables 1, 2 and 3 compare the average, the standard deviation of horn and shunck (original and modified), anandan, singh [1], the algorithm simulated with real wavelet(two cases: our simulation and the result in [7]).

The error estimation of our algorithm is better than other methods. Where the directional property of the complex wavelet make the algorithm able to estimate, with accuracy, many kinds of movement (translation, rotation,..), in Figs. 3, 4 and 5 the complex wavelet is more accurate than the real wavelet. However, filtering the sequence in many direction arise the computational complexity compared of the real filters.

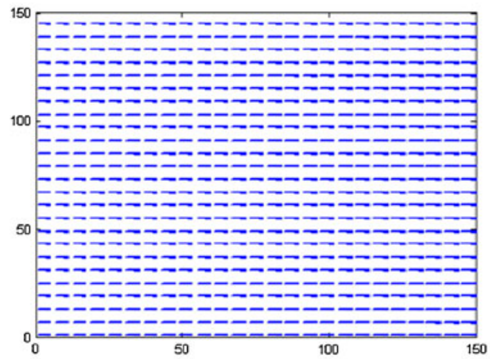
Table 3 Comparison between different methods of sequence « Yosemite».

Methods	Images	Average error	Deviation	Density (%)
Motion estimation using complex wavelet	2	4.52°	4.65°	100
Motion estimation using real wavelet [7]	2	8.43°	10.12°	100
Motion estimation using real wavelet(our simulation)	2	9.43°	8.87°	100
Horn& Schunk (original)	2	32.43 °	30.28°	100
Horn & Schunk (modified)	7–13	11.26°	10.59°	100
Anandan	2	15.84°	13.46°	100
Singh	2	13.16°	12.07°	100

a)

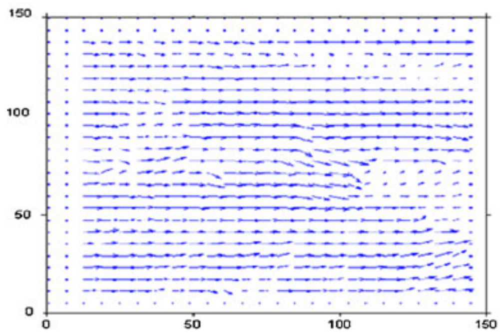
Sequence «Translating Tree»

b)



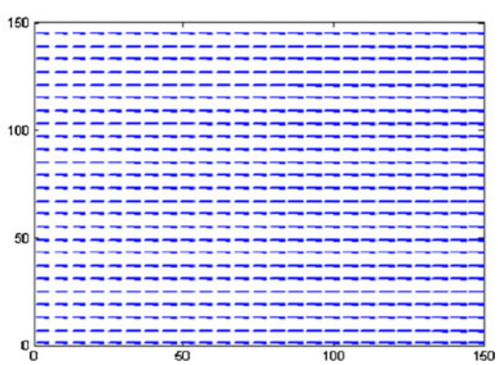
real Optical flow of sequence «Ttranslating Tree»

c)



Optical flow measuring of sequence «Translating Tree»using real wavelet transform

d)



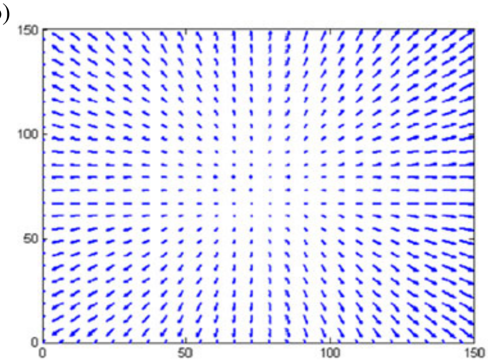
Optical flow measuring of sequence «Translating Tree»using complex wavelet transform

Figure 3 Optical flow estimation of sequence «translating Tree».

a)

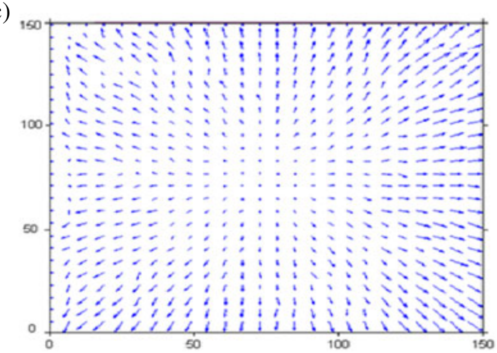
Sequence «Diverging Tree»

b)



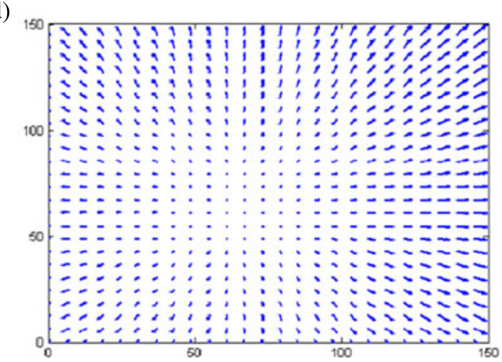
real Optical flow of sequence «DivergingTree»

c)



Optical flow measuring of sequence «Diverging Tree» using real wavelet transform

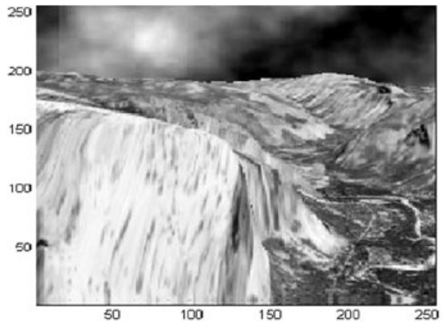
d)



Optical flow measuring of sequence «Diverging Tree» using complex wavelet transform

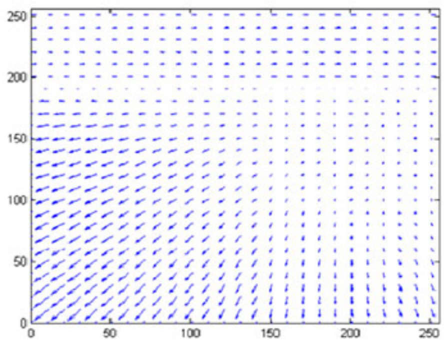
Figure 4 Optical flow estimation of sequence «Diverging Tree».

a)



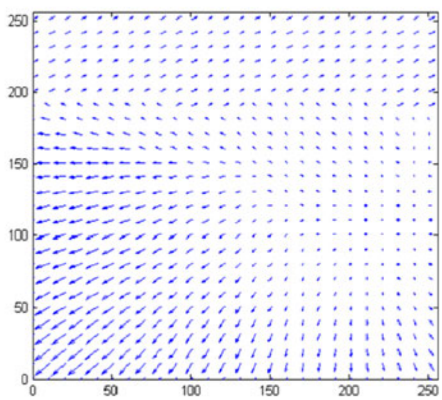
Sequence « Yosemite »

b)



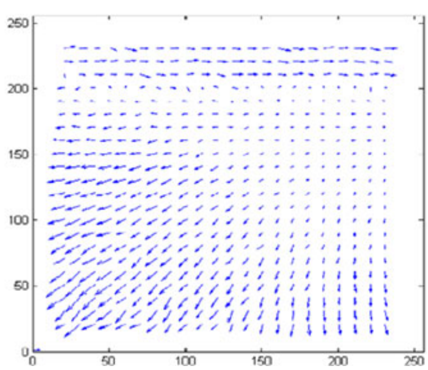
real Optical flowof sequence «Yosemite »

c)



Optical flow measuring of sequence «Yosemite » using real wavelet transform

d)



Optical flow measuring of sequence «Yosemite »using complex wavelet transform

Figure 5 Optical flow estimation of sequence «Yosmite».

4 Conclusion

A fast algorithm for motion estimation is proposed. The algorithm estimate the optical flow by resolving the resulted equations from the projection of the gradient constraint in the Daubechies complex wavelet bases in a coarse and fine manner.

In comparison with the Discrete Wavelet Transform (DWT), the complex wavelet transform possesses two key properties for computer vision: shift invariance, which makes it possible to extract stable local features in an image; and good directional selectivity, making it possible to measure image energy accurately in multiple directions. The experimental results demonstrate that the complex wavelet is capable to estimate many kinds of movement; the proposed method outperforms the method based on real wavelet transform presented in [7]. On the other hand, filtering the sequence with the complex coefficients arise the computational complexity compared of the real filters.

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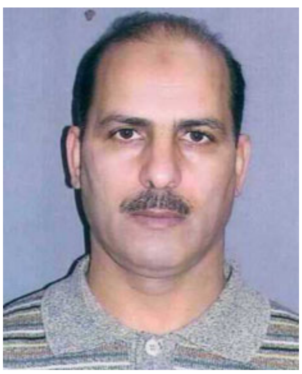

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